Exercise 4C

1 a A 90% confidence interval is

$$\bar{x} \pm 1.645 \times \frac{\sigma}{\sqrt{n}} = 75872 \pm 1.645 \times \frac{15000}{\sqrt{80}}$$
$$= 75872 \pm 2758.75$$
$$= (73113.25, 78630.75)$$
$$= (73113, 78631)$$

b Since n is large, the central limit theorem allows us to approximate the mean distance travelled as a normal distribution and so we can find a confidence interval for the mean distance travelled.

2 a
$$\sigma^2 = \frac{1}{12}[(\mu+10)-(\mu-10)]^2$$
$$= \frac{1}{12}(20)^2$$
$$= \frac{400}{12}$$
$$= \frac{100}{3}$$

b
$$\bar{x} \pm 1.96 \times \frac{\sigma}{\sqrt{n}} = 78.7 \pm 1.96 \times \frac{\sqrt{\frac{100}{3}}}{\sqrt{120}}$$
$$= 78.7 \pm 1.033$$
$$= (77.667, 79.733)$$
$$= (77.7, 79.7)$$

c Since n is large then the distribution of the sample mean will be approximately normally distributed.

3 a
$$\bar{x} \pm 1.96 \times \frac{\sigma}{\sqrt{n}} = 175 \pm 1.96 \times \frac{80}{\sqrt{40}}$$
$$= 175 \pm 24.79$$
$$= (150.21,\ 199.79)$$
$$= (150.2,\ 199.8)$$

b It is not necessary to assume that the value of merchandise sold has a normal distribution because the sample size is large and we can use the central limit theorem.

4 a
$$\bar{x} \pm 1.645 \times \frac{\sigma}{\sqrt{n}} = 14.5 \pm 1.645 \times \frac{1.5}{\sqrt{50}}$$
$$= 14.5 \pm 0.349$$
$$= (14.151, 14.849)$$
$$= (14.15, 14.85)$$

b Maike should suggest that the supermarket reduces the stated fat content since the stated value of 15% is above the confidence interval.

5 a Since the sample size is large, the central limit theorem can approximate the sample mean as a normal distribution as the original distribution is not assumed to be normal.

b $\bar{x} \pm 1.96 \times \frac{\sigma}{\sqrt{n}} = 1.902 \pm 1.96 \times \frac{0.7}{\sqrt{100}}$

$= 1.902 \pm 0.1372$

$= (1.765, 2.039)$

$= (1.77, 2.04)$

6 a $\sigma^2 = \frac{1}{12}\left[(a+8)-(a-4)\right]^2$

$= \frac{144}{12}$

$= 12$

$\mu = \frac{(a-4)+(a+8)}{2}$

$= a+2$

$\bar{Y} \sim \mathrm{N}\left(a+2, \frac{12}{30}\right)$

$\bar{Y} \sim \mathrm{N}(a+2, 0.4)$

b $\bar{Y} \sim \mathrm{N}(a+2, 0.4)$

Therefore a 99% confidence interval for μ is

$\left(\bar{Y} - 2.576 \times \sqrt{0.4}, \bar{Y} - 2.576 \times \sqrt{0.4}\right)$

$= \left(12.6 - 2.576 \times \sqrt{0.4}, 12.6 - 2.576 \times \sqrt{0.4}\right)$

$= (10.97, 14.23)$

Since $\mu = a+2$

the maximum value of Y is $a+8 = \mu+6$

So the confidence interval for the maximum value of Y is:

$(16.97, 20.23)$